

# Table of Z-Transforms

Function or Theorem	Discrete-Time Domain $x[n]$	Z-Transform $X(z) = \mathcal{Z}\{x[n]\}$	ROC
Unit Impulse	$\delta[n]$	1	all $z$
Shifted Impulse	$\delta[n - k]$	$z^{-k}$	all $z$ (except $z = 0$ if $k > 0$ )
Unit Step	$u[n]$	$\frac{1}{1 - z^{-1}} = \frac{z}{z - 1}$	$ z  > 1$
Causal Ramp	$n u[n]$	$\frac{z^{-1}}{(1 - z^{-1})^2} = \frac{z}{(z - 1)^2}$	$ z  > 1$
Causal $n^2$	$n^2 u[n]$	$\frac{z(z + 1)}{(z - 1)^3}$	$ z  > 1$
Real Exponential	$a^n u[n]$	$\frac{1}{1 - az^{-1}} = \frac{z}{z - a}$	$ z  >  a $
Anti-Causal Exponential	$-a^n u[-n - 1]$	$\frac{z}{z - a}$	$ z  <  a $
Exponential $\times$ Ramp	$n a^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2} = \frac{az}{(z - a)^2}$	$ z  >  a $
Exponential $\times n^2$	$n^2 a^n u[n]$	$\frac{az(z + a)}{(z - a)^3}$	$ z  >  a $
$n$ -th Power Times Exponential	$\binom{n}{k} a^{n-k} u[n]$	$\frac{z}{(z - a)^{k+1}}$	$ z  >  a $
Cosine	$\cos(\omega_0 n) u[n]$	$\frac{1 - z^{-1} \cos \omega_0}{1 - 2z^{-1} \cos \omega_0 + z^{-2}}$	$ z  > 1$
Sine	$\sin(\omega_0 n) u[n]$	$\frac{z^{-1} \sin \omega_0}{1 - 2z^{-1} \cos \omega_0 + z^{-2}}$	$ z  > 1$
Exponential Decay Cosine	$r^n \cos(\omega_0 n) u[n]$	$\frac{1 - rz^{-1} \cos \omega_0}{1 - 2rz^{-1} \cos \omega_0 + r^2 z^{-2}}$	$ z  > r$
Exponential Decay Sine	$r^n \sin(\omega_0 n) u[n]$	$\frac{rz^{-1} \sin \omega_0}{1 - 2rz^{-1} \cos \omega_0 + r^2 z^{-2}}$	$ z  > r$
Complex Exponential	$e^{j\omega_0 n} u[n]$	$\frac{z}{z - e^{j\omega_0}}$	$ z  > 1$
Linearity	$ax[n] + by[n]$	$aX(z) + bY(z)$	at least $\text{ROC}_X \cap \text{ROC}_Y$
Time Shift	$x[n - k]$	$z^{-k} X(z)$	same ROC as $X(z)$
Multiplication by $n$	$n x[n]$	$-z \frac{d}{dz} X(z)$	same ROC as $X(z)$
Multiplication by $n^k$	$n^k x[n]$	$\left(-z \frac{d}{dz}\right)^k X(z)$	same ROC as $X(z)$
Scaling in $z$ (Exponential Weighting)	$a^n x[n]$	$X\left(\frac{z}{a}\right)$	$ a  \cdot \text{ROC of } X$
Time Reversal	$x[-n]$	$X(z^{-1})$	1/ROC of $X$
Conjugation	$x^*[n]$	$X^*(z^*)$	same ROC as $X(z)$
Convolution	$x[n] * y[n]$	$X(z) Y(z)$	at least $\text{ROC}_X \cap \text{ROC}_Y$
Correlation	$\sum_{k=-\infty}^{\infty} x[k] y[k - n]$	$X(z) Y(z^{-1})$	at least $\text{ROC}_X \cap 1/\text{ROC}_Y$
Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{X(z)}{1 - z^{-1}}$	$\text{ROC} \cap  z  > 1$
First Difference	$x[n] - x[n - 1]$	$(1 - z^{-1}) X(z)$	$\text{ROC} \cap  z  \neq 0$
Parseval's Theorem	$\sum_{n=-\infty}^{\infty}  x[n] ^2$	$\frac{1}{2\pi j} \oint X(z) X^*(1/z^*) z^{-1} dz$	—
Initial Value Theorem	$x[0]$	$\lim_{z \rightarrow \infty} X(z)$	requires $x[n]$ causal
Final Value Theorem	$\lim_{n \rightarrow \infty} x[n]$	$\lim_{z \rightarrow 1} (z - 1) X(z)$	requires poles inside $ z  < 1$ (except simple pole at $z = 1$ )