

Table of Laplace Transform

Function or Theorem	Time Domain $f(t)$	Laplace Transform $F(s) = \mathcal{L}\{f(t)\}$	ROC
Unit Impulse	$\delta(t)$	1	all s
Shifted Dirac Delta	$\delta(t - \tau)$	$e^{-\tau s}$	all s
Step Function	$u(t - \tau)$	$\frac{e^{-\tau s}}{s}$	$\text{Re}(s) > 0$
Ramp Function	$tu(t)$	$\frac{1}{s^2}$	$\text{Re}(s) > 0$
Shifted Ramp	$(t - a)u(t - a)$	$\frac{e^{-as}}{s^2}$	$\text{Re}(s) > 0$
Exponential	$e^{at}u(t)$	$\frac{1}{s - a}$	$\text{Re}(s) > a$
n -th Power	$t^n u(t)$	$\frac{n!}{s^{n+1}}$	$\text{Re}(s) > 0$
Shifted Power	$(t - a)^n u(t - a)$	$\frac{n! e^{-as}}{s^{n+1}}$	$\text{Re}(s) > 0$
Half-Integer Power	$t^{n - \frac{1}{2}} u(t)$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n - 1) \sqrt{\pi}}{2^n s^{n + \frac{1}{2}}}$	$\text{Re}(s) > 0$
Real Power	$t^p u(t), p > -1$	$\frac{\Gamma(p + 1)}{s^{p+1}}$	$\text{Re}(s) > 0$
Sine	$\sin(\omega t)u(t)$	$\frac{\omega}{s^2 + \omega^2}$	$\text{Re}(s) > 0$
Cosine	$\cos(\omega t)u(t)$	$\frac{s}{s^2 + \omega^2}$	$\text{Re}(s) > 0$
Sine with Phase Shift	$\sin(\omega t + \gamma)u(t)$	$\frac{s \sin \gamma + \omega \cos \gamma}{s^2 + \omega^2}$	$\text{Re}(s) > 0$
Cosine with Phase Shift	$\cos(\omega t + \gamma)u(t)$	$\frac{s \cos \gamma - \omega \sin \gamma}{s^2 + \omega^2}$	$\text{Re}(s) > 0$
Hyperbolic Sine	$\sinh(\alpha t)u(t)$	$\frac{\alpha}{s^2 - \alpha^2}$	$\text{Re}(s) > \alpha $
Hyperbolic Cosine	$\cosh(\alpha t)u(t)$	$\frac{s}{s^2 - \alpha^2}$	$\text{Re}(s) > \alpha $
First Shifting Theorem	$e^{\alpha t} f(t)$	$F(s - \alpha)$	shift ROC right by α : $\text{Re}(s) > \sigma_0 + \alpha$
Time Scaling ($a > 0$)	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$	$\text{Re}(s/a) > \sigma_0$
Exponential Decay Sine	$e^{-\alpha t} \sin(\omega t)u(t)$	$\frac{\omega}{(s + \alpha)^2 + \omega^2}$	$\text{Re}(s) > -\alpha$
Exponential Decay Cosine	$e^{-\alpha t} \cos(\omega t)u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega^2}$	$\text{Re}(s) > -\alpha$
Power with Exponential	$\frac{r}{(k-1)!} t^{k-1} e^{\lambda t}$	$\frac{r}{(s - \lambda)^k}$	$\text{Re}(s) > \text{Re}(\lambda)$
Second Shifting Theorem	$u(t - a)f(t - a)$	$e^{-as} F(s)$	same ROC as $F(s)$
Modified Second Shifting	$u(t - a)f(t)$	$e^{-as} \mathcal{L}\{f(t + a)\}$	shift ROC left by a
Multiplication by t	$tf(t)$	$-\frac{d}{ds} F(s)$	same ROC as $F(s)$
Multiplication by t^n	$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n} F(s)$	same ROC as $F(s)$
Integration in Time	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$	$\text{Re}(s) > \sigma_0$
First Derivative	$f'(t)$	$sF(s) - f(0)$	same ROC as $F(s)$
Second Derivative	$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$	same ROC as $F(s)$
n -th Derivative	$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$	same ROC as $F(s)$
Convolution	$\int_0^t f(\tau)g(t - \tau) d\tau$	$F(s)G(s)$	intersection of the two ROCs
Initial Value Theorem	$\lim_{t \rightarrow 0^+} f(t)$	$\lim_{s \rightarrow \infty} sF(s)$	requires no poles in $\text{Re}(s) > 0$
Final Value Theorem	$\lim_{t \rightarrow \infty} f(t)$	$\lim_{s \rightarrow 0} sF(s)$	requires poles only in $\text{Re}(s) < 0$