Table of Elementary Indefinite Integrals [1]

GENERAL FORMULAS

Let $f = f(x)$, $g = g(x)$ be functions of x, and α, β be constants.

1. $\int [\alpha f + \beta g]\, dx = \alpha \int f\, dx + \beta \int g\, dx$
2. $\int f(x)\, dx = \int f[g(u)] g'(u)\, du$
3. $\int f' g\, dx = fg - \int f g'\, dx$
4. $\int \frac{f'}{f}\, dx = \ln|f| + C$
5. $\int f' g + g' f\, dx = fg + C$
6. $\int \frac{f'g - g'f}{g^2}\, dx = \frac{f}{g} + C$
7. $\int \frac{f'g - g'f}{fg}\, dx = \ln\left|\frac{f}{g}\right| + C$
8. $\int \frac{f'g - g'f}{f^2 + g^2}\, dx = \tan^{-1} \frac{f}{g} + C$
9. $\int \frac{f'g - g'f}{f^2 - g^2}\, dx = \frac{1}{2} \ln\left|\frac{f-g}{f+g}\right| + C$

Let $f(ax+b)$ be a function composition of $ax+b$, then

$$\int f(ax+b)\, dx = \frac{1}{a} F(ax+b) + C,$$

where $F' = f$.

1. $\int (ax+b)^n\, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$
2. $\int \frac{1}{(ax+b)^n}\, dx = \frac{-1}{a(n-1)} \cdot \frac{1}{(ax+b)^{n-1}} + C$

Inverse Function. For an invertible function $f(x)$,

$$\int f^{-1}(x)\, dx = x f^{-1}(x) - F(f^{-1}(x)) + C,$$

where $F' = f$.

ALGEBRAIC FUNCTIONS

Integrals Involving Powers.

1. $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C, \quad [n \neq -1]$
2. $\int \frac{1}{x}\, dx = \ln|x| + C$
3. $\int \frac{1}{x^2}\, dx = -\frac{1}{x} + C$
4. $\int \frac{1}{x^n}\, dx = \frac{-1}{(n-1)x^{n-1}} + C, \quad [n \neq 1]$
5. $\int \sqrt[n]{x}\, dx = \frac{n}{n+1} \cdot x \sqrt[n]{x} + C, \quad [n \neq -1]$
6. $\int \frac{1}{\sqrt[n]{x}}\, dx = \frac{n}{n-1} \cdot \frac{x}{\sqrt[n]{x}} + C, \quad [n \neq 1]$

Integrals of Standard Rational Functions.

1. $\int \frac{1}{x^2 + a^2}\, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$
2. $\int \frac{x}{x^2 \pm a^2}\, dx = \frac{1}{2} \ln|x^2 \pm a^2| + C$
3. $\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$
4. $\int \frac{1}{a^2 - x^2}\, dx = \frac{1}{2a} \ln\left|\frac{x+a}{x-a}\right| + C$
5. $\int \frac{1}{(x-a)(x-b)}\, dx = \frac{1}{a-b} \ln\left|\frac{x-a}{x-b}\right| + C$
6. $\int \frac{x^2}{x^2 + a^2}\, dx = x - a \tan^{-1}\left(\frac{x}{a}\right) + C$
7. $\int \frac{x^2}{x^2 - a^2}\, dx = \frac{a}{2} \ln\left|\frac{x-a}{x+a}\right| + x + C$

Integrals Involving Standard Roots.

1. $\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$
2. $\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}} + C$
3. $\int x\sqrt{x^2 \pm a^2}\, dx = \frac{1}{3}(x^2 \pm a^2)^{3/2} + C$
4. $\int x\sqrt{a^2 - x^2}\, dx = -\frac{1}{3}(a^2 - x^2)^{3/2} + C$
5. $\int \frac{1}{\sqrt{x^2 \pm a^2}}\, dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$
6. $\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1} \frac{x}{a} + C$
7. $\int \frac{x}{\sqrt{x^2 \pm a^2}}\, dx = \sqrt{x^2 \pm a^2} + C$
8. $\int \frac{x}{\sqrt{a^2 - x^2}}\, dx = -\sqrt{a^2 - x^2} + C$
9. $\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\, dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \mp \frac{a^2}{2} \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$
10. $\int \frac{1}{\sqrt{(x-a)(x-b)}}\, dx = \ln\left|x - \frac{a+b}{2}\right| + \sqrt{(x-a)(x-b)} + C$
11. $\int \sqrt{\frac{x}{a-x}}\, dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a} + C$
12. $\int \sqrt{\frac{x}{a+x}}\, dx = \sqrt{x(a+x)} - a \ln(\sqrt{x} + \sqrt{x+a}) + C$

13. $\int \frac{1}{\sqrt{2ax - x^2}}\, dx = \cos^{-1}\left(\frac{a-x}{a}\right) + C$
14. $\int \frac{x}{\sqrt{2ax - x^2}}\, dx = a \cos^{-1}\left(\frac{a-x}{a}\right) - \sqrt{2ax - x^2} + C$
15. $\int \frac{x^2}{\sqrt{2ax - x^2}}\, dx = \frac{3a^2}{2} \cos^{-1}\left(\frac{a-x}{a}\right) - \frac{x+3a}{2} \sqrt{2ax - x^2} + C$
16. $\int \frac{1}{x\sqrt{2ax - x^2}}\, dx = -\frac{\sqrt{2ax - x^2}}{ax} + C$

Integrals Involving $\sqrt{ax+b}$. Let $R = ax + b$.

1. $\int \sqrt{R}\, dx = \frac{2}{3a} R^{3/2} + C$
2. $\int x\sqrt{R}\, dx = \frac{2}{15a^2}(3a^2x^2 + abx - 2b^2)\sqrt{R} + C$
3. $\int \sqrt{xR}\, dx = \frac{1}{4a^{3/2}} \left[(2ax+b)\sqrt{axR} - b^2 \ln\left|a\sqrt{x} + \sqrt{aR}\right|\right] + C$
4. $\int \sqrt{x^3 R}\, dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2 x} + \frac{x}{3}\right] \sqrt{x^3 R} + \frac{b^3}{8a^{5/2}} \ln\left|a\sqrt{x} + \sqrt{aR}\right| + C$
5. $\int \frac{1}{\sqrt{R}}\, dx = \frac{2R}{a} + C$

Integrals Involving Power of 3/2.

1. $\int (a^2 - x^2)^{3/2}\, dx = -\frac{x}{8}(2x^2 - 5a^2)\sqrt{a^2 - x^2} + \frac{3a^4}{8} \sin^{-1} \frac{x}{a} + C$
2. $\int \frac{1}{(x^2 \pm a^2)^{3/2}}\, dx = \pm \frac{1}{a^2} \cdot \frac{x}{\sqrt{x^2 + a^2}} + C$
3. $\int \frac{1}{(a^2 - x^2)^{3/2}}\, dx = \frac{x}{a^2 \sqrt{a^2 - x^2}} + C$
4. $\int \frac{x}{(x^2 \pm a^2)^{3/2}}\, dx = -\frac{1}{\sqrt{x^2 \pm a^2}} + C$

TRIGONOMETRIC FUNCTIONS

First Degree.

1. $\int \sin x\, dx = -\cos x + C$
2. $\int \cos x\, dx = \sin x + C$
3. $\int \tan x\, dx = \ln|\sec x| + C$
4. $\int \cot x\, dx = \ln|\sin x| + C$
5. $\int \csc x\, dx = \ln|\csc x - \cot x| + C = \ln\left|\tan \frac{x}{2}\right| + C$
6. $\int \sec x\, dx = \ln|\sec x + \tan x| + C = \ln\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$

Second Degree.

1. $\int \sin^2 x\, dx = \frac{x}{2} - \frac{1}{2} \sin x \cos x + C$
2. $\int \cos^2 x\, dx = \frac{x}{2} + \frac{1}{2} \sin x \cos x + C$
3. $\int \tan^2 x\, dx = \tan x - x + C$
4. $\int \cot^2 x = -\cot x - x + C$
5. $\int \csc^2 x\, dx = -\cot x + C$
6. $\int \sec^2 x\, dx = \tan x + C$

Third Degree.

1. $\int \sin^3 x\, dx = -\frac{1}{3}(2 + \sin^2 x) \cos x + C$
2. $\int \cos^3 x\, dx = \frac{1}{3}(2 + \sin^2 x) \cos x + C$
3. $\int \tan^3 x\, dx = \frac{1}{2} \tan^2 x + \ln|\cos x| + C$
4. $\int \csc^3 x\, dx = -\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln|\csc x - \cot x| + C$
5. $\int \sec^3 x\, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C$
6. $\int \cot^3 x\, dx = -\frac{1}{2} \cot^2 x - \ln|\sin x| + C$

Recurrence Formula for n-th Degree

1. $\int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x\, dx$
2. $\int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x\, dx$
3. $\int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x\, dx$
4. $\int \csc^n x\, dx = -\frac{\csc^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} x\, dx$
5. $\int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} + \int \tan^{n-2} x\, dx$
6. $\int \cot^n x\, dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x\, dx$

[1] This note was written by Don D. Le.

Combination Integrals.

1. $\int \sec x \tan x \, dx = \sec x + C$
2. $\int \csc x \cot x \, dx = -\csc x + C$
3. $\int \sec^n x \tan x \, dx = \frac{1}{n}\sec^n x + C$
4. $\int \csc^n x \cot x \, dx = -\frac{1}{n}\csc^n x + C$

Product of Trigonometric Functions and Power Functions.

1. $\int x \sin x \, dx = -x\cos x + \sin x + C$
2. $\int x \cos x \, dx = \cos x + x \sin x + C$
3. $\int x^2 \sin x \, dx = (2-x^2)\cos x + 2x \sin x + C$
4. $\int x^2 \cos x \, dx = 2x\cos x + (x^2-2)\sin x + C$

INVERSE TRIGONOMETRIC FUNCTIONS

1. $\int \sin^{-1} x \, dx = x\sin^{-1} x + \sqrt{1-x^2} + C, \quad |x| \le 1$
2. $\int \cos^{-1} x \, dx = x\cos^{-1} x - \sqrt{1-x^2} + C, \quad |x| \le 1$
3. $\int \tan^{-1} x \, dx = x\tan^{-1} x - \frac{1}{2}\ln|1+x^2| + C$
4. $\int \cot^{-1} x \, dx = x\cot^{-1} x - \frac{1}{2}\ln|1+x^2| + C$
5. $\int \csc^{-1} x \, dx = x\csc^{-1} x + \ln\left|x(1+\sqrt{1-x^{-2}})\right| + C, \quad |x| \ge 1$
6. $\int \sec^{-1} x \, dx = x\sec^{-1} x + \ln\left|x(1+\sqrt{1-x^{-2}})\right| + C. \quad |x| \ge 1$

EXPONENTIAL FUNCTIONS

Basic Formulas

1. $\int e^x \, dx = e^x + C$
2. $\int a^x \, dx = \frac{a^x}{\ln a} + C$
3. $\int f' e^f \, dx = e^f + C$
4. $\int e^x[f + f'] \, dx = fe^x + C$
5. $\int \frac{1}{1+e^x} \, dx = \ln\left(\frac{e^x}{1+e^x}\right) + C$
6. $\int \frac{1}{a+be^{mx}} \, dx = \frac{mx - \ln(a+be^{mx})}{am} + C$
7. $\int \frac{e^{mx}}{a+be^{mx}} \, dx = \frac{\ln(a+be^{mx})}{mb} + C$

Exponential Function with Polynomials.

1. $\int xe^{ax} \, dx = e^{ax}\left(\frac{x}{a} - \frac{1}{a^2}\right) + C$
2. $\int x^2 e^{ax} \, dx = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) + C$
3. $\int P_m(x)e^{ax} \, dx = \frac{e^{ax}}{a}\sum_{k=0}^{m}(-1)^k \frac{P^{(k)}(x)}{a^k} + C,$

where $P_m(x)$ is a polynomial of degree m and $P^{(k)}(x)$ is the kth derivative of $P_m(x)$.

Exponential Functions with Trigonometric Functions

1. $\int e^{ax}\sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$
2. $\int e^{ax}\cos bx \, dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + C$
3. $\int e^{ax}\sin^2 bx \, dx = \frac{e^{ax}}{a(a^2+4b^2)}(a^2\sin^2 bx - ab\sin(2bx) + 2b^2) + C$
4. $\int e^{ax}\cos^2 bx \, dx = \frac{e^{ax}}{a(a^2+4b^2)}(a^2\cos^2 bx - ab\sin(2bx) + 2b^2) + C$

LOGARITHM FUNCTION

Integrals Involving $\ln^m(ax)$.

1. $\int \ln x \, dx = x\ln x - x + C$
2. $\int \log_a x \, dx = \frac{x\ln x - x}{\ln a} + C$
3. $\int \ln^2(ax) \, dx = x\ln^2(ax) - 2x\ln(ax) + 2x + C$
4. $\int \ln^3(ax) \, dx = x\ln^3(ax) - 3x\ln^2(ax) + 6x\ln(ax) - 6x + C$
5. $\int \ln^m(ax) \, dx = \frac{x}{m+1}\sum_{k=0}^{m}(-1)^k\frac{(m+1)!}{(m-k)!}\ln^{m-k}(ax) + C$

Integrals Involving $\ln(ax)$ and Powers of x. $[n, m > 0]$

1. $\int x\ln(ax) \, dx = \frac{x^2}{2}\ln(ax) - \frac{x^2}{4} + C$
2. $\int x^2\ln(ax) \, dx = \frac{x^3}{3}\ln(ax) - \frac{x^3}{9} + C$
3. $\int x^n\ln(ax) \, dx = x^{n+1}\left[\frac{\ln(ax)}{n+1} - \frac{1}{(n+1)^2}\right] + C$
4. $\int x^n\ln^2(ax) \, dx = x^{n+1}\left[\frac{\ln^2(ax)}{n+1} - \frac{2\ln(ax)}{(n+1)^2} + \frac{2}{(n+1)^3}\right] + C$
5. $\int x^n\ln^m(ax) \, dx = \frac{x^{n+1}}{m+1}\sum_{k=0}^{m}\frac{(m+1)!}{(m-k)!}\cdot\frac{\ln^{m-k}(ax)}{(n+1)^{k+1}} + C$

Integrals Involving $\ln^m(ax)/x^n$.

1. $\int \frac{\ln(ax)}{x} \, dx = \frac{1}{2}[\ln(ax)]^2 + C$
2. $\int \frac{\ln(ax)}{x^2} \, dx = -\frac{1}{x}[\ln(ax) + 1] + C$
3. $\int \frac{\ln^n(ax)}{x} \, dx = \frac{\ln^{n+1}(ax)}{n+1} + C$
4. $\int \frac{\ln^n(ax)}{x^m} \, dx = \frac{-1}{(n+1)x^{m-1}}\sum_{k=0}^{n}\frac{(n+1)!\ln^{n-k}(ax)}{(n-k)!(m-1)^{k+1}} + C \quad [m > 1]$

Integrals Involving $1/[x\ln^m(ax)]$ $\quad [m > 1]$

1. $\int \frac{dx}{x\ln(ax)} = \ln|\ln(ax)| + C$
2. $\int \frac{dx}{x\ln^m(ax)} = \frac{-1}{(m-1)\ln^{m-1}(ax)} + C$

Integrals Involving $\ln(x^2 \pm a^2)$.

1. $\int \ln(x^2+a^2) \, dx = x\ln(x^2+a^2) + 2a\tan^{-1}\left(\frac{x}{a}\right) - 2x + C$
2. $\int \ln|x^2-a^2| \, dx = x\ln(x^2-a^2) + a\ln\left|\frac{x+a}{x-a}\right| - 2x + C$
3. $\int x\ln(x^2+a^2) \, dx = \frac{1}{2}\left[(x^2+a^2)\ln(x^2+a^2) - x^2\right] + C$
4. $\int x\ln|x^2-a^2| \, dx = \frac{1}{2}\left[(x^2-a^2)\ln|x^2-a^2| - x^2\right] + C$

HYPERBOLIC FUNCTIONS

First Degree.

1. $\int \sinh x \, dx = -\cosh x + C$
2. $\int \cosh x \, dx = \sinh x + C$
3. $\int \tanh x \, dx = \ln(\cosh x) + C$
4. $\int \coth x \, dx = \ln|\sinh x| + C$
5. $\int \operatorname{csch} x \, dx = \ln\left|\tanh\left(\frac{x}{2}\right)\right| + C$
6. $\int \operatorname{sech} x \, dx = \tan^{-1}(\sinh x) + C$

Second Degree of sech and csch.

1. $\int \operatorname{sech}^2 x \, dx = \tanh x + C$
2. $\int \operatorname{csch}^2 x \, dx = -\coth x + C$

Combination Integrals.

1. $\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$
2. $\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$
3. $\int \sinh ax \cosh ax \, dx = \frac{1}{4a}[\sinh 2ax - 2ax] + C$
4. $\int \sinh ax \cosh bx = \frac{1}{a^2-b^2}[a\cosh ax \sinh bx - b\cosh bx \sinh ax] + C$

Hyperbolic Functions with Trigonometric Functions.

1. $\int \cos ax \cosh bx \, dx = \frac{1}{a^2+b^2}[a\sin ax \cosh bx + b\cos ax \sinh bx] + C$
2. $\int \cos ax \sinh bx \, dx = \frac{1}{a^2+b^2}[a\sin ax \sinh bx + b\cos ax \cosh bx] + C$
3. $\int \sin ax \cosh bx \, dx = \frac{-1}{a^2+b^2}[a\cos ax \cosh bx - b\sin ax \sinh bx] + C$
4. $\int \sin ax \sinh bx \, dx = \frac{-1}{a^2+b^2}[a\cos ax \sinh bx - b\cosh bx \sin ax] + C$
5. $\int \sinh ax \cosh bx \, dx = \begin{cases} \frac{1}{b^2-a^2}[b\cosh bx \sinh ax - a\cosh ax \sinh bx] + C, & a \ne b \\ \frac{1}{4a}[\sinh(2ax) - 2ax] + C, & a = b \end{cases}$

Hyperbolic Functions with Exponential Functions.

1. $\int e^{ax}\sinh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2-b^2}[a\sinh bx - b\cosh bx] + C, & a \ne b \\ \frac{1}{4a}e^{2ax} - \frac{x}{2} + C, & a = b \end{cases}$
2. $\int e^{ax}\cosh bx \, dx = \begin{cases} \frac{e^{ax}}{a^2-b^2}[a\cosh bx - b\sinh bx] + C, & a \ne b \\ \frac{1}{4a}e^{2ax} + \frac{x}{2} + C, & a = b \end{cases}$

INVERSE HYPERBOLIC FUNCTIONS

1. $\int \sinh^{-1} x \, dx = x\sinh^{-1} x - \sqrt{x^2+1} + C$
2. $\int \cosh^{-1} x \, dx = x\cosh^{-1} x - \sqrt{x^2+1} + C$
3. $\int \tanh^{-1} x \, dx = x\tanh^{-1} x + \frac{1}{2}\ln(1-x^2) + C, \quad |x| < 1$
4. $\int \coth^{-1} x \, dx = x\coth^{-1} x + \frac{1}{2}\ln(x^2-1) + C, \quad |x| > 1$
5. $\int \operatorname{csch}^{-1} x \, dx = x\operatorname{csch}^{-1} x + |\sinh^{-1} x| + C, \quad x \ne 0$
6. $\int \operatorname{sech}^{-1} x \, dx = x\operatorname{sech}^{-1} x + \sin^{-1} x + C, \quad 0 < x \le 1$